$$\text{D'où } S_n(x) = \sum_{k=0}^{n}[(-1)(x-1)]^k = \frac{1-[(-1)(x-1)]^{n+1}}{1-[(-1)(x-1)]}$$
$$= \frac{1-[(-1)(x-1)]^{n+1}}{x} = \frac{1}{x} - (-1)^{n+1}\frac{(x-1)^{n+1}}{x}$$

b- $S_n(x) = \sum_{k=0}^{n}[(-1)(x-1)]^k = \frac{1}{x} - (-1)^{n+1}\frac{(x-1)^{n+1}}{x}; \forall x > 0$

$$\Rightarrow \int_1^2 \sum_{k=0}^{n}[(-1)(x-1)]^k dx = \int_1^2 \left[\frac{1}{x} - (-1)^{n+1}\frac{(x-1)^{n+1}}{x}\right]dx$$

$$\Leftrightarrow \sum_{k=0}^{n}(-1)^k \int_1^2 (x-1)^k dx = \int_1^2 \frac{1}{x}dx - (-1)^{n+1}\int_1^2 \frac{(x-1)^{n+1}}{x}dx$$

$$\Leftrightarrow \sum_{k=0}^{n}(-1)^k \left[\frac{(x-1)^{k+1}}{k+1}\right]_1^2$$
$$= [\text{Log}(x)]_1^2 - (-1)^{n+1}[\text{Log}2 - (n+1)u_n]$$

$$\Leftrightarrow \sum_{k=0}^{n}(-1)^k \frac{1}{k+1} = \text{Log}2 - (-1)^{n+1}[\text{Log}2 - (n+1)u_n]$$

$$\Leftrightarrow v_n == \text{Log}2 - (-1)^{n+1}[\text{Log}2 - (n+1)u_n]$$

$$\Leftrightarrow \text{Log}2 - v_n = (-1)^{n+1}[\text{Log}2 - (n+1)u_n].$$

3/ $\text{Log}2 - v_n = (-1)^{n+1}[\text{Log}2 - (n+1)u_n]; \forall n \in IN^*$

$$\Rightarrow |\text{Log}2 - v_n| = |(-1)^{n+1}[\text{Log}2 - (n+1)u_n]|; \forall n \in IN^*$$

$$\Leftrightarrow |\text{Log}2 - v_n| = |\text{Log}2 - (n+1)u_n|; \forall n \in IN^*$$

Comme $\lim\limits_{n\to+\infty}[\text{Log}2-(n+1)u_n] = 0$ alors $\lim\limits_{n\to+\infty}|\text{Log}2 - v_n| = 0$

$$\Leftrightarrow \lim_{n\to+\infty} v_n = \text{Log}2.$$